Refined similarity hypothesis and asymmetry of turbulence

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An experimental study of atmospheric turbulence and turbulent pipe flow reveals some deviations from the refined similarity hypothesis (RSH), in addition to those already known. Special emphasis is placed on studying the turbulence asymmetry, as it has proved to be a sensitive indicator of this deviation. It is found that, in spite of good correlation between the velocity increments u and the dissipation field y, the typical amplitude of the latter is appreciably smaller than that of the velocity increments for violent events. In the framework of the RSH, the quantity $V=u/y$ is supposed to be statistically independent of the dissipation. The study shows that there is some weak dependence, manifested at least in a correlation between the sign of V and the amplitude of the dissipation field y. We suggest some modification of the measure in defining the dissipation field, so that the asymmetry can be treated in a self-consistent way in the framework of the RSH.
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I. INTRODUCTION

Self-similar properties of turbulence have been studied for a long time. In particular, Kolmogorov suggested scaling laws for the velocity increment structure functions $\Delta_r v$ (r is the distance), and that is now called $K41$ theory [1]. It was later understood that the theory implies relatively simple statistics. Roughly speaking, the predicted scaling corresponds to a nonintermittent system, and that in turn corresponds to self-similar probability distributions $p(\Delta_r v/\langle(\Delta_r v)^2\rangle^{1/2}|r)$: i.e., this function is independent of r, and is universal.

On the other hand, the intermittency of the turbulence corresponds to a deviation from simple self-similarity. Kolmogorov's refined similarity hypothesis (RSH) does allow the intermittency [2], and therefore the requirements of self-similarity are modified. Indeed, this time, the distribution $p(V|r)$, where $V=\Delta_r v/(r\varepsilon_r)^{1/3}$, is supposed to be self-similar, that is, the distribution is an universal function, in particular independent of r (see also Ref. [3]).

Strictly speaking, the Kolmogorov law [4], found in 1941, already implies that the probability distribution function (PDF) $p(V|r)$ does depend on r; at least, it depends on $r/|r|$. This deviation *a priori* has been considered to be small. Traditionally, this asymmetry was interpreted as a manifestation of turbulent energy cascade to the small scales [5].

It was recently suggested that the asymmetry may be also related to the deviation from self-similarity [6–9]. It became clear that the asymmetry is manifested mainly in the PDF's tails [9], responsible for the intermittency. Connecting the intermittency of the increments with that of the dissipation field y via the RSH, we conclude that the latter should be asymmetric as well (in fact, there are some experimental indications that this is indeed the case [8]). As $y\geqslant 0$, and therefore the sign of the velocity increments is defined by the V field only, the asymmetry for y implies that the value of y ''knows'' about the sign of the V field; that is, there is some correlation between y and the V field. However, this contradicts the RSH, as in its framework these two fields should be statistically independent. Thus the asymmetry properties of turbulence have so far not been treated self-consistently. The aim of the paper is to provide an experimental study of the asymmetry-related deviations from the RSH, i.e., the deviations from self-similarity of the distribution $p(V|r)$, suggesting an attempt and self-consistent treatment of asymmetry in the framework of the RSH.

As, on the other hand, the asymmetry appeared to be related to the intermittency, this study may give some additional insight into the problem of intermittency. In spite of quite intensive recent experimental studies to verify the RSH [10–15], the asymmetry aspects of this problem were not addressed in detail.

Section II is introductory as well, describing the methods used in the paper. The asymmetry aspects of the RSH are given in Sec. III. A brief review of previous experimental results relevant to this problem are given in Sec. IV. Some of the deviations from simple self-similarity of the velocity increments PDF listed in Ref. [9], and in other previous papers, also correspond to the deviations from the RSH, and they are briefly repeated in this section, thus slightly overlapping with Ref. [9]: otherwise the description would not be complete. A comparison of the dissipation field with that of the velocity increments is given in Sec. V. The difference between them is analyzed in Sec. VI. Sections VII and VIII are devoted to an evaluation of the moments of V distribution. It is shown that the moments are functions of the distance r, thus presenting a deviation from the RSH. The odd moments show positive skewness, described in Sec. VIII, thus revealing some correlation between the V field and the dissipation. A possible way to reconcile with the RSH is suggested in Sec. IX. The key point is to modify the dissipation measure. Finally, the main conclusions are given in Sec. X.

II. DESCRIPTION OF THE METHOD AND DENOTATIONS

The measurements of atmospheric turbulence were performed at Yale University. The Taylor microscale Reynolds number is estimated to be 9540. The data set consisting of 10 000 000 data points was divided into four files; below we will refer to them as runs A, B, C, and D. We use Taylor's hypothesis in interpreting the data; that is, the time series is

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treated as a one-dimensional cut of the process (for more detail, see Ref. [9]). We also use the data from a pipe turbulence (from Yale University as well), with 2 000 000 data points, and the Reynolds number is estimated to be 230 000. These data are used only once: just to check the third moment of V distribution (see Sec. VIII).

We study the statistics of velocity increments for different distances r. All the distances are given in terms of Kolmogorov scale η. We measured the velocity increments for 12 distances, uniformly distributed between $r/\eta = 133.3$ and 1600.0. All these distances are well inside the inertial range, the integral scale l being estimated as $l = 86\,671\eta$. That was for the atmospheric turbulence. As to the pipe experiment, the inertial range is shorter (because the Reynolds number is less), and therefore we picked another set of distances, roughly corresponding to that for the atmospheric turbulence. The smallest distance is $r/\eta = 31.6$, and the largest is $r/\eta = 379.8$, the intermediate points being uniformly distributed between these two. We will also use r_0, corresponding to the smallest sample distance: some of the measurements are provided by this.

The dissipation field is defined as usual (see, e.g., Ref. [5]),

$$\varepsilon_r = \frac{1}{r^D} \int \varepsilon(\mathbf{x}) d\mathbf{x}, \tag{1}$$

where D is the dimension of space. As we deal with a one-dimensional cut of the process, we imply $D=1$ in all the formulas; the letter D is kept just for a possible generalization of the formulas used below. The dissipation is also understood as one dimensional (and sometimes called pseudodissipation); that is,

$$\varepsilon = 15\nu(\partial_x v)^2,$$

(ν is the kinematic viscosity), which is relevant for isotropic processes. To be more specific, we write Eq. (1) as follows:

$$\varepsilon_r = \frac{1}{r} \int_{x-r/2}^{x+r/2} \varepsilon(x) dx. \tag{2}$$

According to the refined similarity hypothesis [2], the velocity increment $\Delta_r v = v(x+r) - v(x)$ obeys the equation

$$\Delta_r v = V(\varepsilon_r r)^{1/3} = V(\langle\varepsilon\rangle r)^{1/3} y, \quad y = \left(\frac{\varepsilon_r}{\langle\varepsilon\rangle}\right)^{1/3}, \tag{3}$$

where V is a random function statistically independent of ε. In addition, the PDF for V is supposed to be a universal function. In a somewhat more relaxed form, this universality is understood as the independence of the distance r, as well as the independence of the dissipation [3]. Let us write Eq. (3) in dimensionless form,

$$u = CVy, \tag{4}$$

where

$$u = \frac{\Delta_r v}{\langle(\Delta_r v)^2\rangle^{1/2}}$$

and

$$C = \frac{(\langle\varepsilon\rangle r)^{1/3}}{\langle(\Delta_r v)^2\rangle^{1/2}}. \tag{5}$$

In a fairly good approximation, the coefficient C is a constant, and $C = 1/C_2^{1/2}$, where C_2 is the Kolmogorov constant, $C_2 = 2 \pm 0.4$. In these calculations, however, the intermittency correction is included, that is,

$$\langle(\Delta_r v)^2\rangle = C_2(\langle\varepsilon\rangle r)^{2/3}\left(\frac{r}{l}\right)^{-\mu_2} \tag{6}$$

[3]. As $\mu_2 = -0.04$, the coefficient C is strictly speaking not a constant, depending weakly on r:

$$C = \frac{1}{C_2^{1/2}}\left(\frac{r}{l}\right)^{-0.02}. \tag{7}$$

Therefore, we will study the distribution function for

$$V = \frac{u}{Cy}. \tag{8}$$

The central part of a PDF, within three standard deviations, we call the "core." Everything beyond this we will call "tails," provided the values there are anomalous: either substantially exceeding Gaussian values, or anomalously depending on r. The distributions containing tails we call "singular." We also recall that the tails usually correspond to the intermittency. At least, that is the case for the velocity increments PDF's.

We will consider both structure functions,

$$\mathcal{S}_n(r) = \langle\Delta_r v^n\rangle \sim r^{\xi_q}, \tag{9}$$

where n is a positive integer, and generalized structure functions,

$$S_q(r) = \langle|\Delta_r v|^q\rangle \sim r^{\zeta_q} = r^{q/3-\mu_q}, \tag{10}$$

where the intermittency corrections μ_q are defined by the dissipation field

$$\langle\varepsilon_r^{q/3}\rangle \sim r^{-\mu_q = -(D-D_{q/3})(q/3-1)}, \tag{11}$$

and D_q are the generalized dimensions [16].

We can write the joint PDF's

$$p(u,y|r) = \frac{1}{Cy}\, p_V\left(\frac{u}{Cy}\right) p_y(y|r)$$

or

$$p(u,V|r) = p_V(V)\, \frac{1}{C|V|}\, p_y\left(\frac{u}{CV}\middle| r\right).$$

As

$$\langle u^n\rangle = C^n\langle V^n\rangle\langle y^n\rangle \quad \text{and} \quad \langle|u|^q\rangle = C^q\langle|V|^q\rangle\langle y^q\rangle,$$

we have

$$\mathcal{S}_n(r)=S_2(r)^{n/2}\int u^n P(u,y|r)du\,dy,$$

$$S_q(r)=S_2(r)^{q/2}\int |u|^q P(u,y|r)du\,dy,\tag{12}$$

$$S_2=\int(\Delta_r v)^2 P(\Delta_r v,y|r)d\Delta_r v\,dy.$$

It is clear from Eq. (12) that the $K41$ scaling [1] corresponds to the case when ε_r is treated as a constant; that is, $\langle y^q\rangle$ is independent of r, or $p_y(y|r)=\delta(y-1)$. More generally, the $K41$ scaling is recovered if $p_y(y|r)$ is a function of y only. Indeed, in that case,

$$\mathcal{S}_n(r)=\mathcal{C}_n(\langle\varepsilon\rangle r)^{n\zeta_1},\quad S_q(r)=C_q(\langle\varepsilon\rangle r)^{q\zeta_1}.$$

It follows from the Kolmogorov law [4] that, in the inertial range, and for $n=3$,

$$\mathcal{S}_3(r)=-\tfrac{4}{5}\langle\varepsilon\rangle r,\tag{13}$$

and therefore, $\zeta_1=\frac{1}{3}$, recovering $K41$.

In the framework of the RSH, $p_y(y|r)$ might be a function of both y and r, and it is assumed that all moments $\langle y^q\rangle$ are scaling $\sim r^{-\mu_q}$. Therefore,

$$\mathcal{S}_n(r)=\mathcal{C}_n\left(\langle\varepsilon r\rangle^{n/3}\left(\frac{r}{l}\right)^{-\mu_n},\quad \mathcal{S}_q(r)=C_q(\langle\varepsilon r\rangle^{q/3}\left(\frac{r}{l}\right)^{-\mu_q}.\right.\tag{14}$$

Thus, in the framework of the RSH, the PDF $p_y(y|r)$ is not just a function of r. The distribution is supposed to be *singular*. In particular, the flatness,

$$F(r)=\frac{S_4(r)}{S_2(r)^2}\sim\left(\frac{r}{l}\right)^{-(D-D_{4/3})/3-(D-D_{2/3})2/3},\tag{15}$$

is larger than, say, the Gaussian value (=const=3), and, as seen from Eq. (15), is a power law. As the exponent is negative, this function is growing with decreasing distance r, and it is maximal at the Kolmogorov scale η. Large flatness means, in turn, that the velocity increments are intermittent. In this case, the PDF for both the velocity increments and for y, that is, for the dissipation field, should contain tails. Thus the intermittency corrections appear in the framework of the RSH only due to the intermittency of the dissipation field y, and hence, the PDF $p(V|r)$ is not expected to have any tails. Indeed, the PDF was suggested to be Gaussian, with some small deviations due to asymmetry [11]. For this reason, in this paper we focus mainly on the tails of the PDF's.

It is obvious from Eq. (14) that the scaling exponents are the same both for structure functions, and for generalized structure functions: the difference is only in prefactors $\mathcal{C}_n$ and C_q (and they might be different for odd n). In particular, $\zeta_3=\xi_3=1$, that is, the intermittency correction μ_3 vanishes.

III. RSH WITH EMPHASIS ON ASYMMETRY

It is clear from Eq. (12) that, because y is non-negative, all odd moments of the structure functions should vanish if the PDF $p_V(V)$ is an even function. However, it follows from the Kolmogorov law (13) that the third order structure function does not vanish. There is also strong experimental evidence that all other odd moments also do not vanish [17,6]. All this implies that $p_V(V)$ is asymmetric. On the other hand, the PDF obviously obeys the rule

$$p_V(-V|-r)=p_V(V|r),$$

and therefore, if the PDF is independent of r, then $p_V(-V)=p_V(V)$; that is, it should be symmetric. Since the real PDF is not symmetric [see Eq. (13)], we conclude that it should have some dependence on r: at least, it should depend on the sign of r. That is,

$$p(V|r)=p_v(V,\gamma)\equiv p_V(\gamma V)\quad\text{where}\quad\gamma=\frac{r}{|r|},\tag{16}$$

cf. Eq. (16) in Ref. [9]. Thus, even if the RSH is satisfied, the PDF $p(V|r)$ does depend on r, although weakly: it ''knows'' only about the sign of r. In principle, however, the PDF can be a universal (asymmetric) function, in the spirit of the third Kolmogorov hypothesis. The aim of this paper is to verify if the PDF depends on r only weakly, as in Eq. (16).

We will also consider positive and negative velocity increments separately. That means, we consider

$$S_q^+(r)=\int_0^\infty(\Delta_r v)^q p(\Delta_r v|r)d\Delta_r v,$$

$$S_q^-(r)=\int_{-\infty}^0|\Delta_r v|^q p(\Delta_r v|r)d\Delta_r v.$$

Then, analogously to Eq. (12), we have

$$S_q^\pm(r)=S_2(r)^{q/2}\langle V^q\rangle^\pm\langle y^q\rangle,$$
$$\langle V^q\rangle^+=\int_0^\infty V^q p_V(V)dV,\quad\langle V^q\rangle^-=\int_{-\infty}^0|V|^q p_V(V)dV.\tag{17}$$

Obviously, in the framework of the RSH, each $S_q^+(r)$ function has the same scaling as $S_q^-(r)$, and they both have the same scaling as $S_q(r)$: only the prefactors are different. In addition, $S_3^\pm\sim r$; that is, they both possess the scaling of the Kolmogorov law.

We denote u^+ and u^--positive and negative increments, correspondingly. These are functions of x and r. We denote corresponding dissipation fields by $y^\pm$;

$$y^+=\left(\frac{1}{r}\int_{x-r/2}^{x+r/2}\frac{\varepsilon(x|u^+\neq 0)}{\langle\varepsilon\rangle}dx\right)^{1/3},$$

$$y^-=\left(\frac{1}{r}\int_{x-r/2}^{x+r/2}\frac{\varepsilon(x|u^-\neq 0)}{\langle\varepsilon\rangle}dx\right)^{1/3}.$$

According to Eq. (17),

$$\frac{S_q^-(r)}{S_q^+(r)}=\frac{\langle(u^-)^q\rangle}{\langle(u^+)^q\rangle}=\frac{\langle V^q\rangle^-}{\langle V^q\rangle^+}=\text{const.}\tag{18}$$

The structure functions of a low order, or in particular, of zeroth order, can be studied in detail, because of good statistics. It follows from Eq. (17) that

$$S_0^+(r)=\int_0^\infty p_V(V)dV, \quad S_0^-(r)=\int_{-\infty}^0 p_V(V)dV, \quad (19)$$

so that these two structure functions should be constant.

Finally, we consider flatness for positive and negative distributions separately, which is, in the framework of the RSH,

$$F^\pm(r)=\frac{S_4^\pm(r)}{S_2^\pm(r)^2}\sim\left(\frac{r}{l}\right)^{-(D-D_{4/3})/3-(D-D_{2/3})2/3}. \quad (20)$$

Comparing with Eq. (15), we see that the r dependence of $F^\pm(r)$ should coincide with that of $F(r)$ (although the prefactors could be different).

We note that all asymmetry effects should disappear as the distance becomes large. That is, when r approaches l, the asymmetry is expected to decrease, and for $r\geqslant l$ it should vanish. Indeed, for large distances, the increment is a sum of two independent variables, and therefore the statistic becomes independent of r, and, in particular, independent of the sign of r; see, e.g., Ref. [9]. This means that all odd moments vanish, and the ratio in Eq. (18) becomes unity. In particular,

$$S_0^\pm\to\tfrac{1}{2} \quad \text{at } r\geqslant l. \quad (21)$$

We now summarize the information concerning expected properties of the distributions. The K41 theory corresponds to self-similar PDF for the velocity increments,

$$p(\Delta_r v|r)\Rightarrow P\left(\frac{\Delta_r v}{\langle\Delta_r v^2\rangle^{1/2}}\right),$$

i.e., the PDF can be written as a function of one variable, instead of two ($\Delta_r v$, and r); see, e.g., Ref. [9]. This would correspond to nonintermittent turbulence. In the RSH, the intermittency is allowed, and now the distribution for V is supposed to be universal; that is,

$$p\left(\frac{\Delta_r v}{(\varepsilon_r r)^{1/3}}\middle| r\right)\Rightarrow P_V(V),$$

apart from a weak r dependence, as in Eq. (16). This function $P_V(V)$ is not expected to possess any tails, and is expected to be Gaussian-like, although not exactly, because of needed asymmetry.

The major difference between K41 and the RSH is allowing the intermittency in the latter. In spite of the fact that the difference is of vital importance, some of the corollaries listed in this and previous sections, following from the RSH, coincide with those that follow from requirements of self-similarity of the PDF for the velocity increments. For instance, as seen from Eq. (18), the ratio of the positive and negative parts of the increments should be constant; this is also true for self-similar PDF's [9]. In other words, some of the statistical properties are identically the same both for the K41 theory and for the RSH. In Sec. IV, we will have to repeat in part the analysis given in Ref. [9], concerning the comparison of these corollaries with experimental results, adding to it a more complete interpretation.

IV. EXPERIMENTAL VALIDATION: BRIEF REVIEW OF PREVIOUS RESULTS

Experimental studies and direct numerical simulations testing the RSH started only recently [10–15]. Basically, there are strong supports for the RSH, both experimentally and numerically. However, some deviations from the hypothesis were found as well, and we discuss these below, mostly in the next sections.

We are more interested in asymmetry-related matters, and, as the asymmetry is manifested mainly in the tails of the PDF's, we are going to focus on these. The first prediction following from Eq. (14), that the exponents for the odd order structure functions and generalized structure functions coincide, generally fails: this statement holds only for the third order; otherwise, for the higher orders, ξ_n is systematically larger than ζ_n [6]. This trend was explained in Ref. [6] in the framework of the so-called ramp model. Further studies of plus-minus structure functions, $S_q^\pm(r)$, revealed that their ratios are not constant, in violation of Eq. (18); see Fig. 3 in Ref. [7]. We note two types of deviations from a constant in this figure. First, there is a trend of decreasing (increasing) ratio for $q>1$ ($q<1$) with increasing distance. Second, there are large bumps on the curves. The first trend seems to be expected: all curves should approach unity for large distances; see the end of Sec. III. However, that should be true *outside* the inertial range, at very large distances, whereas the trend is observed inside the range. On the other hand, if a quantity substantially exceeds unity at small distances, as in Fig. 3 in Ref. [7] for $q>1$, and it should asymptotically approach unity at large distances, then one would naturally expect that this quantity would monotonously decrease ''as soon as possible.'' In other words, it would be surprising to find a quantity that is strictly constant, substantially exceeding unity all over the inertial range, and then decreasing rapidly at $r\leqslant l$. As the real quantity does follow ''common sense,'' i.e., does decrease in inertial range, it suggests an idea that the weak violation of the RSH—that is, that $p(V|r)$ does depend on r as in Eq. (16)—is indeed not as ''innocent'' as it seems. Apparently, this violation, although *a priori* seemingly weak, might result in substantial deviations from the RSH. This trend of decreasing (increasing) ratio with growing r can be easily explained in the framework of the ramp model [6]. Indeed, if we suppose that $S_q^+(r)$ and $S_q^-(r)$ scale differently, so that

$$D_q^-<D_q^+, \quad (22)$$

then the ratio should decrease for $q>1$, and increase for $q<1$, in both cases approaching unity: exactly as in Fig. 3 of Ref. [7]; see also Fig. 2(a) in Ref. [9].

Another comparison with the RSH is provided by the measurements of the flatness. It is found that, indeed, the flatness decreases with r, as in Eq. (15); however, the flatness for positive and negative distributions does not follow Eq. (20): see Fig. 1(a) in Ref. [9]. That is, the r dependence is different for $F^+(r)$ and $F^-(r)$, and these two functions differ from $F(r)$.

We now return to the second type of deviation from a constant observed in Fig. 3 of Ref. [7], namely, that the curves are not smooth. This in turn returns us to the discussion of a more general observation that the odd moments usually exhibit poor scaling, and quite scattered data; see, e.g., Ref. [9]. As a well known example, we mention the third moment $S_3(r)$. Indeed, the scaling for the Kolmogorov law [Eq. (13)] is usually worse than the scaling for $S_3(r)$, in spite of the fact that the Kolmogorov law should be satisfied *a priori*, while $S_3(r) \sim r$ scaling is an assumption. The simplest explanation is that the core of the PDF $p(\Delta_r v|r)$ (that is, about 99% of events) is approximately symmetric; see, e.g., Figs. 5 and 6 in Ref. [8]. Therefore, the overwhelming majority of events makes a contribution to the generalized structure functions $S_q(r) = S_q^+(r) + S_q^-(r)$, providing a good statistic, whereas the odd moments do not vanish $[\mathcal{S}_n = S_n^+(r) - S_n^-(r) \neq 0]$ due only to the asymmetry supplied by the tails, that is, only by 1% of events. Still, this explanation is at least incomplete. The thing here is that the plus-minus structure functions also do not behave well, and definitely worse than $S_q(r)$ [9]. If we write $S_q^{\pm}(r) = \tilde{S}_q^{\pm}(r) + n_q^{\pm}(r)$, where $\tilde{S}_q^{\pm}(r)$ corresponds to the "true" scaling, as in Ref. [14], and $n_q^{\pm}(r)$ is a noise, appearing for one or another reason, then we might expect that the noises are statistically independent for the plus and minus distributions. Then the error propagation would result in a summing up of the noises for these two distributions, and therefore $S_q(r) = S_q^+(r) + S_q^-(r)$ would behave worse than $S_q^+(r)$ and $S_q^-(r)$ separately. As this is definitely not the case, it is apparent that $n_q^+(r)$ and $n_q^-(r)$ are correlated, and therefore they should not be called "noises," but rather "fluctuations." Indeed, as observed in Ref. [8] for the lowest moment $S_0^{\pm}(r)$, even a slightest increase in $n_0^+(r)$ is accompanied by decrease of $n_0^-(r)$, and vice versa, so that these two plots for $S_0^+(r)$ and for $S_0^-(r)$ are mirror symmetric (see Fig. 2 in Ref. [8]). This can be explained as well: if the length of the accelerated part the ramp happens to be larger (smaller) than usual, then it may happen only at the expense of the decelerated lag of the ramp, so that the latter has to be smaller (larger). As a result, the fluctuations are canceled for the sum $S_q(r) = S_q^+(r) + S_q^-(r)$, while they remain the same for each of the structure functions $S_q^+(r)$ and $S_q^-(r)$; they sum up for the ratio (18), giving the worst scaling. All this behavior of the moment ratios points to substantial contributions of the PDF tails to the asymmetry.

As follows from Eq. (19), the structure functions of zeroth order should be constant in the inertial range. On the other hand, according to Eq. (21), they should approach $\frac{1}{2}$ when r approaches the integral scale. However, since the values $S_0^{\pm}(r_0)$ are different, and they differ substantially from $\frac{1}{2}$; the curves of $S_0^{\pm}(r)$ do follow common sense. That is, they vary monotonously with distance, only asymptotically approaching $\frac{1}{2}$; see Fig. 2(a) in Ref. [8]. The second trend of increased fluctuations [as compared with $S_0(r)$] is also noticeable in this figure.

V. ANALYSIS OF THE DISSIPATION FIELD TAILS

As we saw, the key to understanding both intermittency corrections and asymmetry lies in the tails of the PDF for the velocity increments, and therefore we consider them here in more detail. The tail increments for $r/\eta = 266.7$ from run A are depicted in Fig. 1(a). It can be seen that the peaks of $|u|$ practically coincide with those of the corresponding dissipation field y, although the amplitude of the latter falls short of that of the increments. To pursue this more quantitatively, we plot the relevant correlations in Fig. 1(b). We see that the correlations $\rho(y^+, u^+)$, and $\rho(y^-, u^-)$ are quite high. Recall that the correlation coefficient is defined as

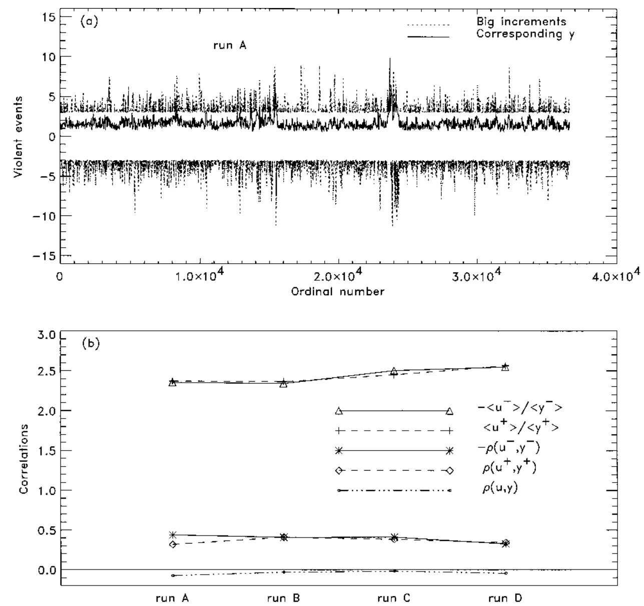

FIG. 1. (a) Depicted are all the tail events, that is, with $|u| \geq 3$, and corresponding dissipation field y, from the run A. It can be seen that $u^{\pm}$ are well correlated with $y^{\pm}$, although the peaks of y are substantially smaller. This is confirmed by (b), where these correlations are depicted for all runs. The ratios $\langle u^{\pm} \rangle / \langle y^{\pm} \rangle$, also depicted, are indeed substantially larger than unity. The u-y correlation is negative, although small. All curves on this plot correspond to the tails of the velocity increment PDF for $r/\eta = 266.7$.

$$\rho(a,b) = \frac{\langle (a - \langle a \rangle)(b - \langle b \rangle) \rangle}{\sqrt{\langle (a - \langle a \rangle)^2 \rangle} \sqrt{\langle (b - \langle b \rangle)^2 \rangle}}.$$

The correlation coefficient (typically ≈ 0.5) can indeed been considered as high, because, if we calculate the coefficient for random numbers with the same number of elements (about 120 000 in all four runs), this coefficient would be two orders of magnitude less. It can be seen that typically the $\rho(y^-, u^-)$ correlation is slightly better than $\rho(y^+, u^+)$. It is also apparent from the figure that, in spite of this good correlation, the typical value of y is substantially less than that of $|u|$.

Further insight into this comparison gives Fig. 2. Here the PDF $p(y|r)$ is directly compared with the positive and negative parts of the velocity increments PDF's, for different distances. On the other hand, this PDF is compared with two nonsingular PDF's, which would appear if the distribution of the velocity gradient $\omega = \partial_x v(x)$ is Gaussian. In the latter case,

$$p(\omega) = \frac{1}{\sqrt{2\pi}} e^{-\omega^2/2}$$

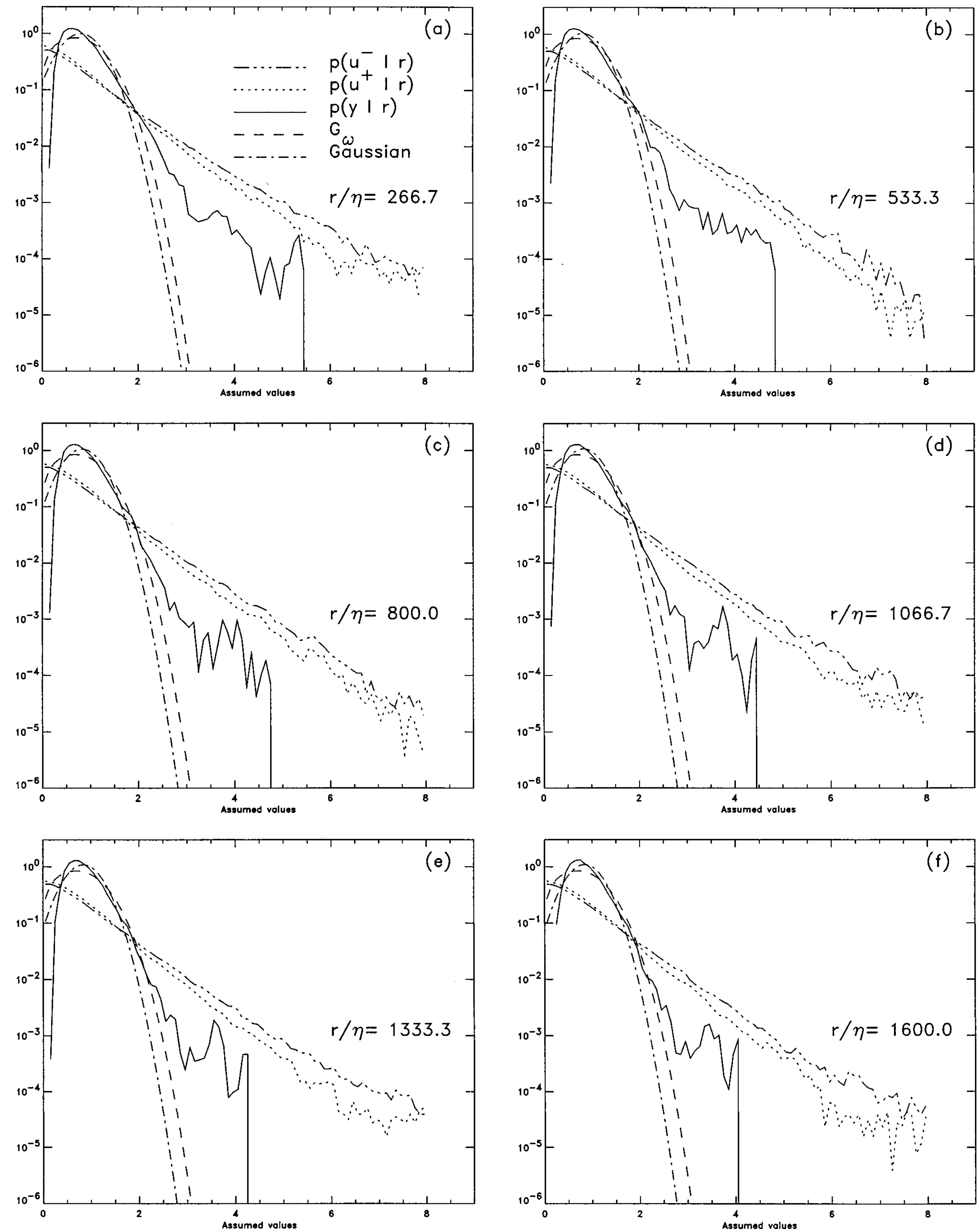

FIG. 2. Probability densities extracted from run A. The PDF's for the dissipation field y are compared with these for the positive and negative velocity increments, $p(u^+|r)$, and $p(u^-|r)$. The distances r are given in terms of Kolmogorov length η, while the assumed values are expressed in terms of $|u|$. G_ω stands for a distribution with Gaussian $\omega = \partial_x v$. Finally, we compare the y distribution with the Gaussian, centered at $\langle y \rangle$, and with variance $\langle y^2 \rangle$-$\langle y \rangle^2$.

(ω is now normalized on its rms), and therefore one of these two distributions, corresponding to the smallest distance r_0, would read

$$p(y|r_0)=G_\omega(y)=\frac{3y^{1/2}}{\sqrt{2\pi}}\,e^{-y^{3}/2}. \tag{23}$$

The second nonsingular distribution should appear at large distances. Indeed, the autocorrelation $\langle\omega(x+r)\omega(x)\rangle$ decreases substantially on one Taylor microscale length λ. Therefore, on distances substantially exceeding λ, measure (2) would present a sum of independent variables, and therefore its distribution is normal as well:

$$p(y|r>\lambda)=\frac{1}{\sqrt{2\pi\langle[y-\langle y\rangle]^2\rangle}}\,e^{-[y-\langle y\rangle]^2/2\langle[y-\langle y\rangle]^2\rangle}. \tag{24}$$

Thus, if ω is Gaussian, then the PDF $p(y|r)$ is a function which lies in between these two distributions (23) and (24). Any real distribution substantially exceeding these two at, say, $y\geqslant 3$, we can call a ''tail,'' and, if the tail exists, we can refer to this distribution as ''singular.'' These two distributions happen to be quite close to each other for all considered distances, as seen from Fig. 2, so that the comparison with a nonsingular distribution is straightforward. It is obvious from the figure that the dissipation field distribution is singular indeed for all these distances. This fact is well known, however. Indeed, if the ω distribution is Gaussian, then the autocorrelation $\langle\varepsilon(x+r)\varepsilon(x)\rangle$ would decrease with distance even faster than the correlation $\langle\omega(x+r)\omega(x)\rangle\sim r^{-4/3}$, namely, $\langle\varepsilon(x+r)\varepsilon(x)\rangle\sim r^{-8/3}$ [18]. However, the real correlation falls off much slower, $\langle\varepsilon(x+r)\varepsilon(x)\rangle\sim\langle\varepsilon_\tau^2\sim r^{-\mu}$, where $\mu=0.25\pm0.05$; see, e.g., Ref. [3]. The exponent μ is called the intermittency exponent, because it is directly related to the generalized dimension D_2, namely, $D_2=1-\mu$ [19]; see also Ref. [20].

Thus the dissipation field is singular. However, as seen from Fig. 2, it is ''not singular enough.'' Indeed, the tails, although they do exist, present some events up to $y=4-5$, and, after that, no events were observed (although the PDF was constructed up to $y=8$), while there was a number of events with $|u|>5$. This feature corresponds, of course, to the plots in Fig. 1. In addition, as seen from Fig. 2, the tails at $y\geqslant 3$ are still below the tails for the velocity increments: they are approximately one order of magnitude less.

Note the main trend of the dissipation field: the tails decrease with increasing distance. This, of course, is expected, because the role of the intermittency decreases with growing r, and this was observed before; see, e.g., Fig. 3 from Ref. [12].

VI. PDF FOR V

We saw in Secs. IV and V that the dissipation field is well correlated with the velocity increments, although it falls short of accounting for large increments, corresponding to the tails. As seen from Fig. 1(b), the values of the large velocity increments are approximately 2.5 time greater than corresponding values of y. The situation is aggravated by the fact that, in order to account for the RSH in form (4), the values of y should be *larger* than the increments in $\sqrt{C_2}\approx\sqrt{2}$ times.

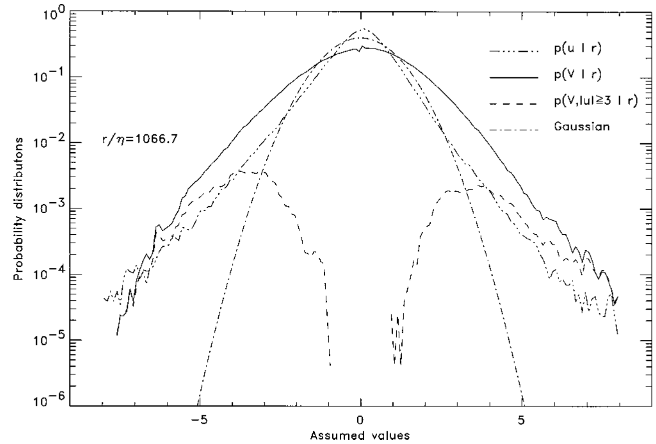

FIG. 3. The PDF's from the data extracted from run A. The tails for $p(V|r)$ distribution are clearly above the tails for the velocity increments PDF.

To see that the V distribution is indeed not satisfactory, we show one example of the PDF for V in Fig. 3. Obviously, there are tails in the PDF for V: this is guaranteed because the PDF $p(V|r)$ is *above* the velocity increments PDF's. Also depicted is $p(V,|u|\geqslant 3|r)$, the tail part of the PDF. It behaves as a conditional PDF for the tails $p(V||u|\geqslant 3,r)$ [that is, for events depicted in Fig. 1(a)], but normalized in such a way that it can fit the plot: otherwise, the PDF should be 70 times higher. Indeed, $p(V,|u|\geqslant 3|r)=p(V||u|\geqslant 3,r)p(|u|\geqslant 3|r)$ and $p(|u|\geqslant 3|r)\approx\frac{1}{70}$. We can see that, again, the tails of the V distribution are higher than these for the increments. This picture is typical of all distances given in Fig. 2, and is not repeated here. Of course, the large values of increments, $|u|>3$, say, are not matched by equally large values of y, as we saw above. Therefore, their ratio, defining V, is still large, and that forms the tails of the PDF.

The singularity of the V distribution does not make sense in the framework of the RSH, and therefore, not violating the spirit of the RSH, we may as well write

$$\Delta_r v=\sqrt{C_2}V_n(\varepsilon_r r)^{1/3} \tag{25}$$

instead of Eq. (3). The new variable V_n,

$$V_n=\frac{\Delta_r v}{\sqrt{C_2}(\varepsilon_r r)^{1/3}},$$

[cf. Eq. (8)] seems to be more ''natural.'' Indeed, the expected variance $\langle V^2\rangle=C_2\approx 2$ (the experimental value is even larger, about 2.2), while the variance for V_n is expected to be about unity. The RSH in dimensionless form is still given by Eq. (4), but the value of C is now close to unity,

$$C=\left(\frac{r}{l}\right)^{-0.02}.$$

With this value of C, the values of y in Eq. (4) should be comparable-to—and not larger than—the increments.

Figure 4 depicts the PDF's for V_n. Here the PDF for $|V_n|\geqslant 3$ is typically below that for the velocity increments.

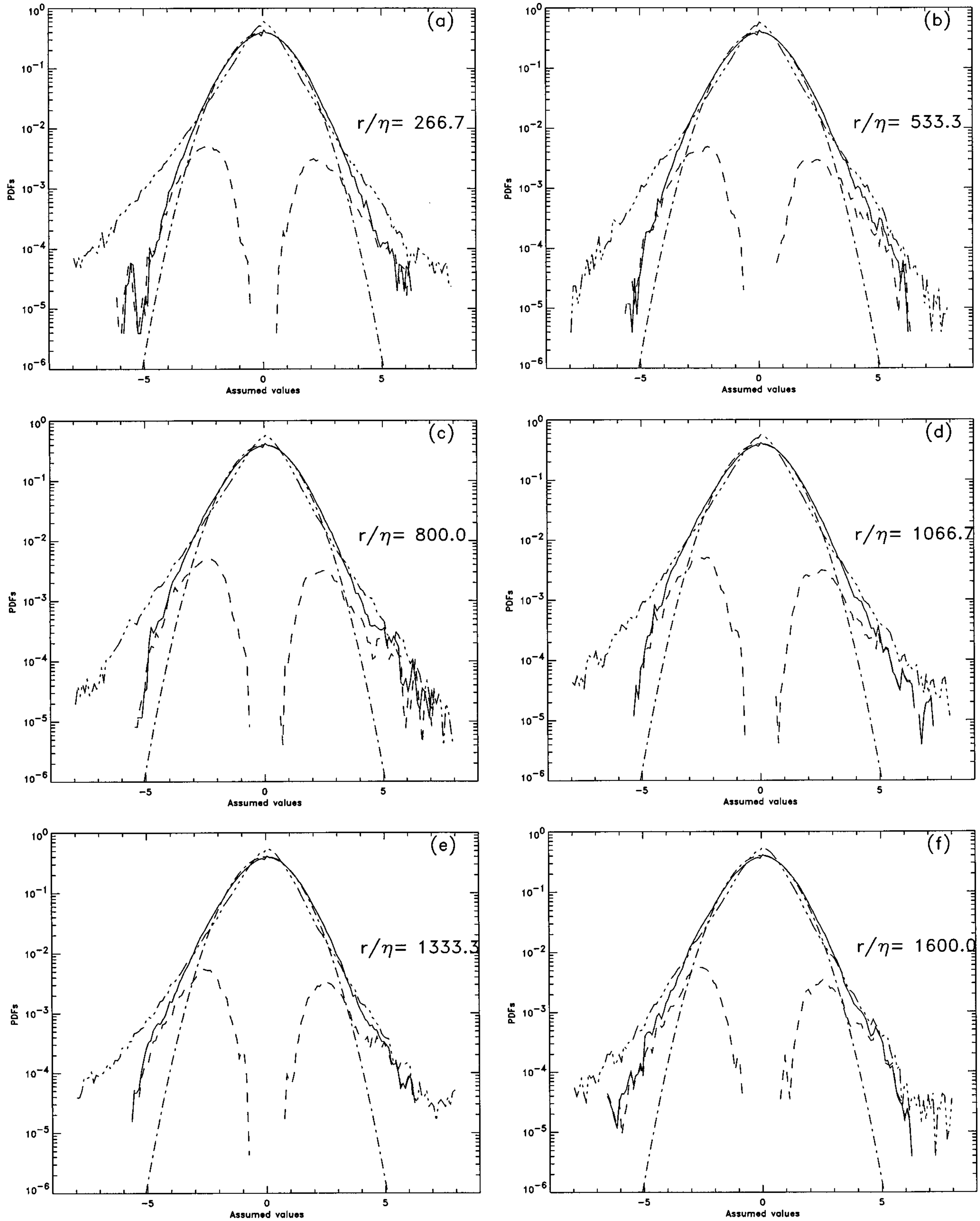

FIG. 4. The PDF's for V_n from run A. The denotations are the same as in Fig. 3.

This is especially true for the left wings (that is, negative values), which could be due to the fact that the left wings of the velocity increments PDF's are always higher than the right wings: due to the asymmetry of the PDF. Surprisingly, the V_n PDF definitely exhibits opposite asymmetry; that is, the right-hand tails are higher than the left-hand ones. This observation is indeed confirmed quantitatively, and we will return to it in Sec. VIII. One can claim for now that there are no tails for negative values of the V_n distribution. However, this statement is certainly not true for the right-hand wings, which practically coincide with the velocity increments distribution.

The most important feature of $p(V_n|r)$ is that it becomes wider with growing r, at least in the region we studied. This is apparent from the figure, and it will be confirmed quantitatively in Secs. VII and VIII. This trend, though, was already apparent from Fig. 10 of Ref. [10(e)], and Fig. 1 of Ref. [15]. Thus, the PDF for V is not self-similar.

Some support for this statement was provided earlier [11], where the conditional PDF $p(V|\varepsilon_r, r)$ was shown to depend on ε_r in inertial range, instead of being independent, as required by the RSH. It can be seen from Figs. 3(a) and 3(b) of Ref. [11(a)], and Fig. 7(b) of Ref. [11(b)], that the innermost curves, that is, the PDF's with the smallest variance, correspond to the lowermost values of ε_r, and vice versa. Presumably, this can be explained as follows. Recall that the correlation coefficients in Fig. 1(b), as well as the ratio $\langle |u| \rangle / \langle y \rangle$, were calculated for the tail part of the PDF. According to all previous experimental results [10–15], the correlation is as good for the whole PDF. On the other hand, the ratio $\langle |u| \rangle / \langle y \rangle$ is not as high for the main distribution; it is about 1.25 in our estimations, as opposed to 2.5 for the tail part; see Fig. 1(b). We may expect that the low values of ε_r will correspond to the velocity increment distribution core, whereas the higher values of ε_r will correspond to the tails. As we saw, the typical value of $|u|/y$ becomes large only in the tails: therefore, the PDF $p(V|\varepsilon_r, r)$ is not expected to have any tails for small ε_r, while it would have them for larger values of ε_r. This results in increasing variance with growing ε_r, as observed in Ref. [11].

VII. EVALUATION OF THE EVEN MOMENTS

In principle, one can normalize Eq. (8) by increasing the constant C: one can redefine the RSH as, say,

$$\Delta_r v = C' V_n (\varepsilon_r r)^{1/3},$$

where $C' > \sqrt{C_2}$, cf. Eq. (25). That would decrease the tails, and the PDF for V_n could go even below the Gaussian distribution for large $|V_n|$. Generally, any transformation

$$V \rightarrow \frac{V_n}{C'},$$

with $C' > 1$ would indeed result in a decrease of the variance $\sim 1/C'^2$. However, the PDF is transformed as well,

$$p(V|r) \rightarrow C' p(V_n C', r),$$

so that the variance

$$\langle V_n^2 \rangle = \frac{\langle V^2 \rangle}{C'^2}.$$

Therefore, if $\langle V^2 \rangle$ is a function of r, then $\langle V_n^2 \rangle$ depends on r in the same way, within the constant $1/C'^2$.

The variance is indeed a function of the distance r, and that can be seen from Fig. 5. The variance $\langle V_n^2 \rangle$ grows with r in the range of distances considered here, and this is true for all four runs. For comparison, we also plot the cumulative variance

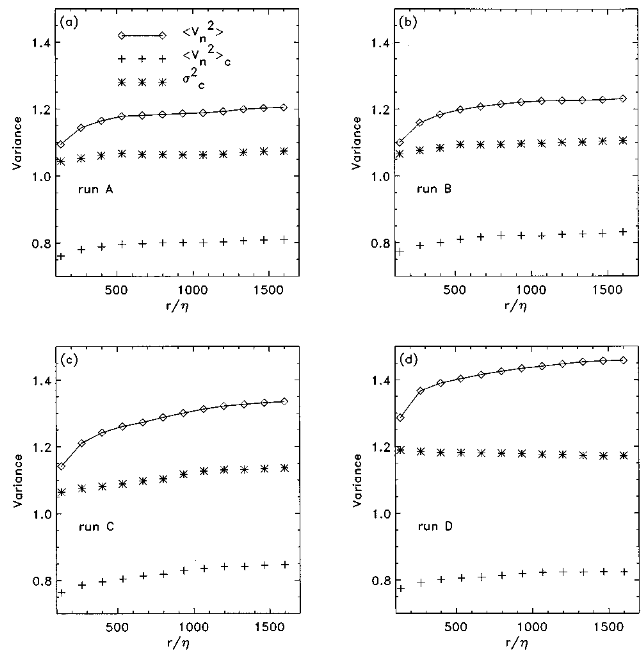

FIG. 5. The variance $\langle V_n^2 \rangle$ proves to be a function of distance r, and it grows even faster than the cumulative second moment σ_c^2. On the other hand, the cumulative moment $\langle V_n^2 \rangle_c$ does not seem to change much.

$$\sigma_c^2 = \langle u^2 \rangle_c = \int_{-3}^{3} u^2 p(u|r) du.$$

Recall that $\langle u^2 \rangle = 1$ by definition, and therefore the cumulative moment is less than unity. If the PDF $p(u|r)$ has no tails (no intermittency), then the cumulative variance would be a constant, i.e., independent of r, although this constant is less than unity. The fact that this variance does depend on r corresponds to the existence of the tails, which are of decreasing intensity with growing distance: therefore the variance grows and asymptotically approaches unity [8,9]. The variance $\langle u^2 \rangle_c$ can be seen to increase with distance in Fig. 5 in all four runs; however, this r dependence is less pronounced than that for the variance $\langle V_n^2 \rangle$. Another comparison is made on this figure with cumulative moment

$$\langle V_n^2 \rangle_c = \int_{-3}^{3} V_n^2 p(V_n|r) du,$$

which does not seem to grow systematically, and for run D this moment even decreases with r. This might explain why the r dependence is hard to detect if one measures the V_n PDF for values of $|V_n|$ that are not large enough. Thus the r dependence of the variance should be attributed only to the parts of the PDF where $|V_n| > 3$, and therefore these parts can still be called "tails", in spite of the fact that they can be hidden by choosing the normalization constant C' sufficiently large (see the beginning of this section).

More evidence about the tails, which make an increasing contribution as the distance increases, is that the flatness also grows with r, as seen from Fig. 6. This trend of flatness growing with distance was observed earlier in Fig. 8(d) of Ref. [10(e)]. Of course, for very large distances, comparable to the integral scale l, $\varepsilon_r \rightarrow \langle \varepsilon \rangle$, and the PDF $p(V|r)$ would

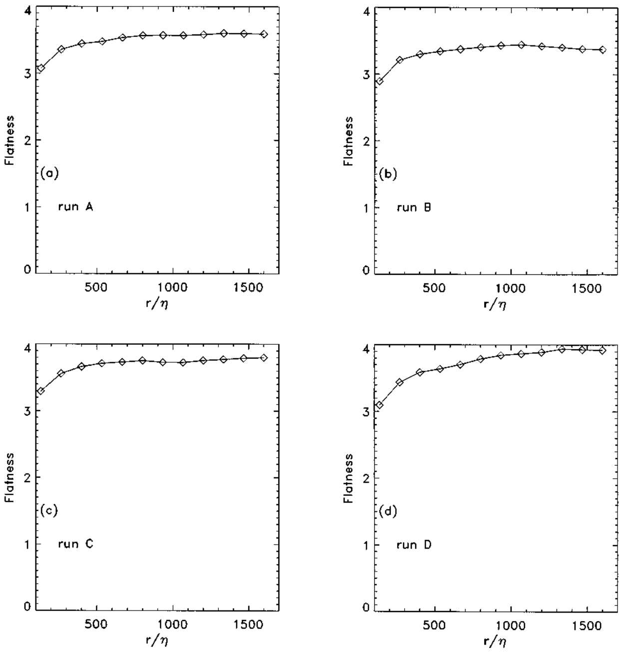

FIG. 6. Flatness $\langle V_n^4 \rangle / \langle V_n^2 \rangle^2$, for all runs.

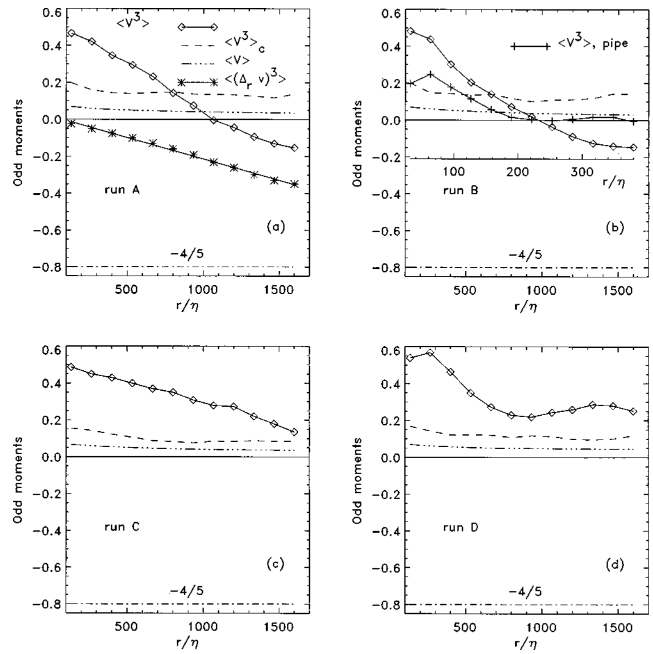

FIG. 7. The first and third moments of V. The first moment is always positive, and it is slowly decreasing with distance. The third moment is typically positive. In order to check the sign of the moments, the third moment for the velocity increments is also depicted. For illustrative purposes, the plot of the latter is normalized in such a way as to fit the figure, and therefore the units for it are arbitrary. One can see that the moment obeys the Kolmogorov law: the plot indeed looks like a straight line, and the values are negative. The cumulative third moment behaves much more smoothly, and it is always positive. The third moment for V distribution from the pipe turbulence is also added for comparison in panel (b); as the ranges of the distances are different (see Sec. II), the scale is also shown on this panel.

approach a one-point velocity distribution which is close to Gaussian [9]. That is to say, both $\langle V_n^2 \rangle$ and the flatness will decrease at that point at least, and will eventually approach the Gaussian values. These two trends—an initially increasing variance, as in Fig. 5, and then a decreasing variance—were observed earlier; see Fig. 8(b) of Ref. [10(e)]. The presence of these two trends might explain why sometimes only flatness decreasing with distance is observed (as in Fig. 4 of Ref. [14]), or sometimes just varying flatness (as in Fig. 3(b) of Ref. [15]). In any case, the V moments deviate from a constant, in violation of the RSH. We finally note that the range chosen in this paper is still far from approaching l. The latter is estimated in Sec. II, and this explains why we observe only one trend of increasing flatness.

It is known that intermittency (and, in particular, flatness) decreases with r, and therefore this behavior of V_n is counterintuitive at least. Close inspection of Fig. 2 shows, however, that the tails of the PDF for u decrease only slightly with r in this range. Only careful study of cumulative moments, etc., results in this conclusion. On the other hand, the tails for the y distribution, already being quite below these of the velocity increments even on the smallest distance [panel (a)], are decreasing quite noticeably. Indeed, the cutoff of the y PDF occurred [see panel (a)] at $y=5.5$, while it is about 4 as seen in panel (f). In fact, in the latter panel, the dissipation field distribution is hardly singular. Thus the trend of decreasing tails for u distribution is less pronounced than the decreasing of the singularity for the y distribution. Roughly, $\langle |u| \rangle$ stays constant, while $\langle y \rangle$ falls off relatively quickly in this range. As a result, the distribution of u/y becomes more singular with increasing distance. This explains why the moments, and also the flatness, grow in this scale range, according to Fig. 6.

VIII. ODD MOMENTS AND ASYMMETRY

As we already noted in Sec. VI, the PDF for V is asymmetric, with an asymmetry opposite to that of the velocity increments. As a matter of fact, even the first moment does not vanish, and it is positive, as seen in Fig. 7. The positive first moment was observed earlier; see Fig. 8(a) of Ref. [10(e)]. This is consistent with the negative correlation $\rho(u,y)$, seen in Fig. 1(b). Indeed, in the framework of the RSH, the V and y fields are statistically independent, and, therefore,

$$0=\langle u \rangle = C\langle V \rangle \langle y \rangle. \qquad (26)$$

As y is non-negative, the quantity $\langle V \rangle$ should vanish. Therefore,

$$\rho(u,y) \sim \langle u\,\delta y \rangle = C\langle V \rangle \langle (\delta y)^2 \rangle = 0,$$

where $\delta y = y - \langle y \rangle$ However, this correlation is observed to be negative. Therefore, the V and y fields are correlated, and we have to write

$$0=\langle u \rangle = C\langle Vy \rangle = C\{\langle V \rangle \langle y \rangle + \langle \delta V\,\delta y \rangle\}$$

instead of Eq. (26), and $\delta V = V - \langle V \rangle$. Therefore,

$$\langle \delta V\,\delta y \rangle = -\langle V \rangle \langle y \rangle.$$

As V is a quickly fluctuating quantity (analogous to u), and y is much more smooth, we can present V as

$$V = \langle V\rangle + \delta V' - \langle V\rangle \frac{\langle y\rangle \delta y}{\langle \delta y^2\rangle},$$

where $\delta V'$ is quickly fluctuating part, uncorrelated with the y field. Thus,

$$\langle u\,\delta y\rangle = C\langle V\rangle \left\{ -\langle y\rangle^2 - \frac{\langle y\rangle\langle \delta y^3\rangle}{\langle \delta y^2\rangle} + \langle \delta y^2\rangle \right\}. \qquad (27)$$

Now, the measured third moment $\langle \delta y^3\rangle$ is positive. No wonder: the quantity δy is limited from below, $\delta y \geqslant -\langle y\rangle$, while it can assume any positive value. Moreover, the fluctuations δy are relatively small, so that, typically, $\langle \delta y^2\rangle \approx \langle y\rangle^2/4$ [this actually can be seen from Fig. 1(a)], and therefore the only positive term in the braces [Eq. (27)]—the last one—is small. Hence, the expression in braces is negative, and thus Eq. (27) explains why $\langle V\rangle > 0$, as in Fig. 7, provided the $\langle u\,\delta y\rangle$ correlation is negative; see Fig. 1(b). Comparison with other studies at this point is somewhat ambiguous. In some papers, a correlation $\rho(u,\varepsilon_r = y^3)$ is studied, rather than $\rho(u,y)$, as above, and it is not clear if the former can also be expected to be negative, like the latter. It is indeed negative in Fig. 5 of Ref. [12], while it is positive in Fig. 2 of Ref. [11(a)], as well as in Fig. 12 of Ref. [10(e)]. It is noteworthy, however, that this correlation does not vanish anyway; that is to say, the correlation coefficient is a smooth function of r, and does not change sign for different distances, and is usually statistically significant. Indeed, in spite of its values [small compared with, say, $\rho(|u|,y)$], it is still quite large compared with the coefficient for random numbers with the same number of elements.

The positive value of the first moment manifests the trend mentioned above in and that the PDF for V is asymmetric, and the right-hand wings are higher than the left-hand ones. This becomes even more evident by studying odd moments of higher orders. Indeed, the third moment is predominantly positive in Fig. 7. Sometimes it becomes negative, although it is still substantially above the Kolmogorov value $-\frac{4}{5}$. This result seems to be puzzling, because other studies gave negative values of $\langle V^3\rangle$ (although with some exceptions; see the discussion below in Sec. X). We therefore obtained the moment by two different means. One recovered the value of $\langle V^3\rangle$ using the PDF's for V. The other one calculated this moment simply by constructing the array u/y directly from the data files. Both methods resulted in almost coinciding plots for $\langle V^3\rangle$ (Fig. 7 depicts this moment calculated from direct construction). In order to make sure that the signs are correct, we processed the third moment of the velocity increments $\langle \Delta_r v^3\rangle$ as well, within the same code that calculates the PDF's and the moments for V distribution. As an illustration, this is shown on Fig. 7(a). It is negative, of course, and decreases linearly with distance, as it should, in accord with the Kolmogorov law (13). Still another test is to construct a cumulative third moment, covering about 98% of events (less than 99% quoted above in Sec. IV, because the variance is now about 2, and we still consider events not exceeding three standard deviations of u), and therefore reliable. The curves indeed look smoother than the full moment, and the values are always positive. Finally, the third moment $\langle V^3\rangle$ from the pipe turbulence data (calculated in a direct way) is given in Fig. 7(b) for comparison. One can say that, qualitatively, the moment behaves the same way as in run B.

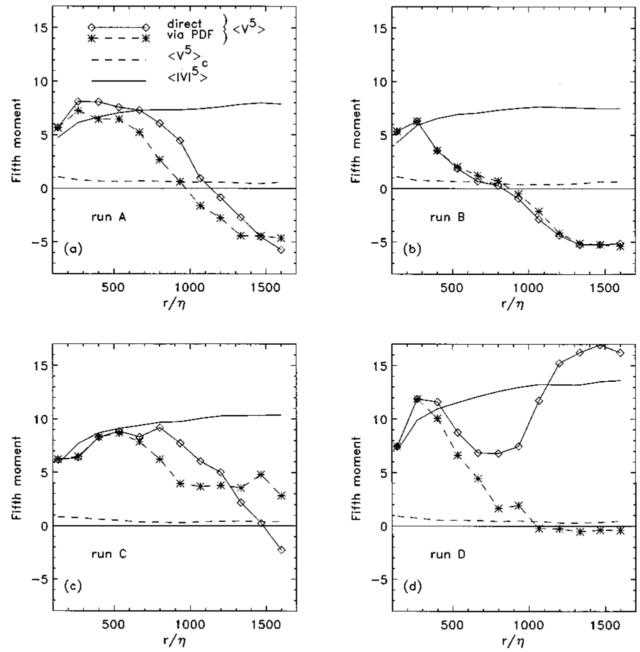

FIG. 8. The fifth moment $\langle V^5\rangle$. We depicted the moment obtained in two ways: both through the PDF $p(V|r)$, constructed above, and by direct calculation of this moment from the data files. The generalized structure function of the fifth order behaves analogously to the variance: it increases with r, cf. Fig. 5. Finally, the cumulative fifth moment behaves analogously to the third one, and it is positive as well.

Further confirmation of this asymmetry, opposite to that of the velocity increments, comes from the fifth moment, see Fig. 8. It is mostly positive. Recall that $\langle \Delta_r v^5\rangle$ is found to be negative, as is the higher odd moments [17], because the negative tails for the velocity increments PDF's are higher than positive [9]. Analogously to the third moment, the fifth one was calculated in two different ways, and both of them are depicted in Fig. 8 for comparison. It can be seen that there is some difference between these two curves, and sometimes the difference is substantial, as for run D. This only means that excitations beyond those taken by the PDF (recall that the PDF was constructed for $|V| \leqslant 8$) play a role for the fifth moment at least, again emphasizing the role of the tails. The cumulative fifth moment behaves much more smoothly, and it is always positive. Finally, $\langle |V|^5\rangle$ behaves in a ''normal'' way, like e.g., the variance in Fig. 5. That is, it grows with distance. Another feature becomes apparent from Fig. 8, namely, that the curves show quite a large scattering of data, and one may say that the convergence is poor. Indeed, these four curves from all runs look quite different. This circumstance points again to the tails of the V distribution: they correspond to rare events, and therefore exhibit strong fluctuations.

As mentioned above, this ''wrong'' asymmetry clearly indicates that there is a correlation between V and y fields; at least, the dissipation field y, being non-negative, ''knows'' if V is positive or negative. We suggest some tentative explanation for this asymmetry, opposite to that of the velocity increments. We first recall that, in the framework of the

RSH, the asymmetry of the velocity increments should result in a corresponding asymmetry of the V field, as in Eq. (16). As the PDF $p(V|r)$ is unrelated to the intermittency (that is, it is not supposed to possess any tails), this asymmetry should be manifested at the core. This means that the contribution to the odd moments of V should come mainly from the core. However, as we saw in Fig. 7, the third moment is formed mainly by the tails: because the cumulative moment is much less than the full one. The same is true for the fifth moment: as seen from Fig. 8, the cumulative moment is negligible. Direct inspection of the PDF core (Fig. 9) leads to the same conclusion. Indeed, while the widening of $p(u|r)$ with growing r, or a decrease of the maximum, is noticeable, as in Fig. 5 from Ref. [8] (although here we consider a substantially smaller distance range than in Ref. [8]), nothing seems to happen to the PDF core of $p(V_n|r)$, although the odd cumulative moments do decrease slightly with distance. In other words, if there is any asymmetry in the core, it is small, and, in addition, it is opposite to that of the velocity increments. We also saw that the tails of $p(V|r)$ are asymmetric in a ''wrong'' way as well, and, therefore, they cannot account for the ''right'' asymmetry of the velocity increments. In other words, the large positive values of V^+ exceed these of $|V^-|$, opposite to the case for the increments: $|u^-|>u^+$. As V is defined by Eq. (8), we conclude that, typically,

$$y^- > y^+ \tag{28}$$

for large y. This means that the negative dissipation field distribution is ''more singular'' than the positive one, in accord with the ramp-model assumption (22). Thus the ramp model follows from the observed ''wrong'' asymmetry of the V distribution.

We now turn this statement around. That is, if the statistics for $y^\pm$ distributions are the same, then both u and V fields would be asymmetric in the same way, having negative skewness. If, however, inequality (28) is satisfied—that is, y^+ and y^- have different statistics, the latter distribution being more singular—then the asymmetry of the V distribution is reduced (because V is inversely proportional to y), thus reducing the skewness. As a matter of fact, this skew, opposite to the velocity increment skewness, *outweighs* the latter, resulting in a nonvanishing first moment and positive odd moments. Thus one may suggest that the ramp model explains the existing asymmetry of the V field.

Recall, however, that both negative and positive dissipation field singularities do not suffice to account for the RSH: they both are ''not singular enough,'' as mentioned at the end of Sec. VII. This explains why the moment $\langle|V|^5\rangle$ behaves much the same way as the even moments in Fig. 5: it grows with distance; see Fig. 8.

There is another quite persistent trend in Figs. 7 and 8: the odd moments *decrease* with distance. Even the pipe turbulence data follow this trend. The only exception can be found in Fig. 8(d) for the fifth moment calculated directly. In this case, however, the data are quite scattered, and therefore less reliable. This trend can be explained as follows. As the singularity of the dissipation field decreases with distance, the V-field statistics become more and more similar to the velocity increment statistics: if the dissipation field is not singular, then the statistics of these two fields would be essentially the same. Therefore, as the distance increases, the values of the odd moments (properly normalized, of course) approach these of the velocity increments; that is, they approach negative values. Due to the ''wrong'' asymmetry of $p(V|r)$, the odd moments are positive at small distances; however, as the singularity of the dissipation field decreases, the odd moments ''try'' to become negative, and therefore, typically, they decrease with distance.

IX. TOWARD A POSSIBLE RECONCILIATION WITH THE RSH

As mentioned, velocity increments are strongly fluctuating, while the dissipation field, being smoothed out by averaging (2), changes much more slowly. Therefore, quite a substantial level of correlation $\rho(u,y)$, reported in all studies, looks quite amazing. It definitely provides a strong support for the RSH. As the fluctuations of the y field are supposed to be responsible for the intermittency, manifested by the velocity increment tails, a direct comparison of violent events of the velocity increments with corresponding fluctuations of the dissipation field, given in Fig. 1, and also showing a high correlation between these two fields, provides an additional and quite convincing argument in favor of the RSH. On the other hand, two deviations from the theory are observed as well. These deviations can be summarized as follows.

(i) The V distribution has tails, responsible for r dependence of the moments of V.

(ii) The V and y fields slightly correlate, resulting in a ''wrong'' asymmetry of the V distribution.

Presumably, the appearance of the tails, point (i), can be attributed to the fact that, although the dissipation field is well correlated with the velocity increments, it falls short of explaining the magnitude: for violent events, $u^\pm/y^\pm \approx 2.5$; see Fig. 1(b). That is to say that the dissipation field ''is not singular enough.'' Apparently, point (ii) can be interpreted to mean that the singularities of the $y^\pm$ fields are different.

In order to treat the asymmetry in a more self-consistent way, we may want to modify measure (2), as in Ref. [21]. That is, let

$$\mu^\pm(x,r) = \int_{x-r/2}^{x+r/2} \varepsilon(x)^{m^\pm} dx,$$

so that the mean measure reads

$$\mu_r^\pm = \frac{1}{r}\,\mu^\pm(x,r), \tag{29}$$

cf. Eq. (2).

If $m^\pm > 1$, then the strength of the singularity increases. Indeed, if D_q are the generalized dimensions for measure (2), then the new generalized dimensions based on measure (29), $D_q^{(m)}$, can be expressed through the old ones,

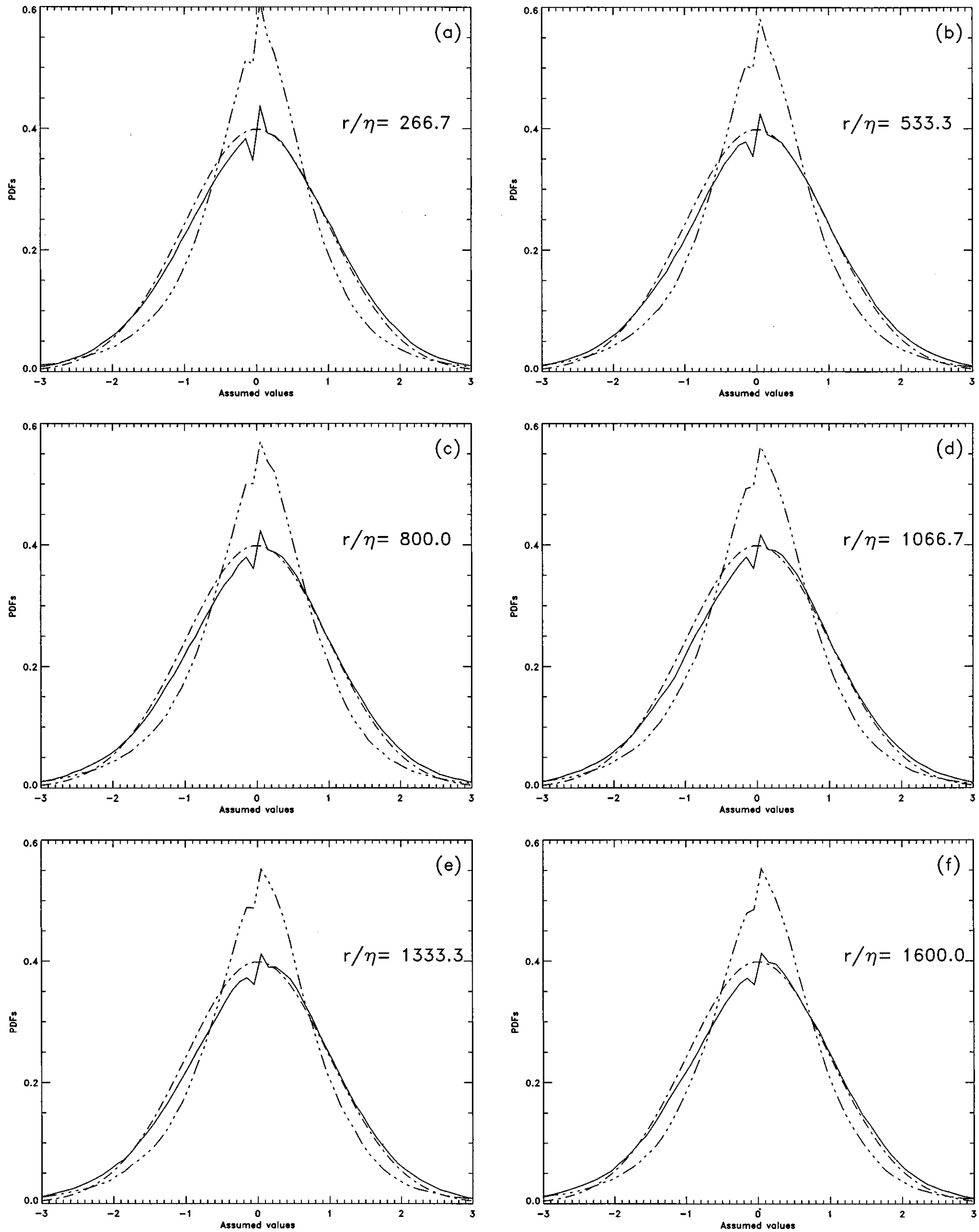

FIG. 9. The PDF's $p(V_n|r)$ and $p(u|r)$ from run A are compared with the Gaussian distribution. The curves are denoted the same way as in Figs. 3 and 4. The only difference with Fig. 4 is that here only the cores of the PDF's are depicted.

$$D_q^{(m)}(q-1)=D_{mq}(mq-1)-D_m(m-1)q; \qquad (30)$$

see formula (3.2) from Ref. [20], adopted from Ref. [22]. It follows from Eq. (30) that, asymptotically,

$$D_\infty-D_\infty^{(m)}=(D_m-D_\infty)(m-1).$$

As the expression in the first parentheses is always positive [23], we have,

$$D_\infty - D_\infty^{(m)} = \begin{cases} >0 & \text{if } m>1 \\ \leqslant 0 & \text{otherwise.} \end{cases}$$

Thus, indeed, the singularity strength increases with increasing m, and therefore we suppose that

$$m^\pm > 1. \tag{31}$$

The measure satisfying Eq. (31) should decrease the gap between the level of the violent evens provided by the velocity increments on one side, and by the dissipation field, on the other, as in Figs. 1 and 2. That should remove the tails from the V distribution, because the large values of $V = u^\pm/(Cy^\pm)$ would be brought to the PDF core by transforming them into

$$\frac{u^\pm}{C(y^\pm)^{m^\pm}}.$$

If, on the other hand, $m^+ = m^-$, then one may expect that the asymmetry of the V distribution might remain ''wrong,'' that is, opposite to the velocity increments. In order to balance this, we suggest that

$$m^+ > m^- > 1. \tag{32}$$

One may expect that Eqs. (31) and (32) would ensure that the V_n distribution has no tails; that is, its PDF is independent of the distance r, and its asymmetry is such that the third moment is negative.

Some indirect support for the modification of the measure is provided by a formula obtained for the ω measure (when $m^\pm = \frac{1}{2}$),

$$\mu^\omega = \int_{x-r/2}^{x+r/2} |\omega| dx, \quad \omega_r = \frac{1}{r^D}\mu^\omega.$$

Then

$$S_q(r) \sim r^{(1-\kappa)q-(D-D_q^\omega)(q-1)}, \tag{33}$$

where κ is so-called cancellation exponent [20]. This exponent is unambiguously defined by the Kolmogorov law; then the RSH can be written in terms of the ω measure,

$$\Delta_r v^\pm = \sqrt{C_2} V_n (\langle \varepsilon \rangle r)^{1/3} \langle \omega_r^3 \rangle^{-1/3} \omega_r ; \tag{34}$$

cf. Eq. (25). Expression (33) was obtained *without* invoking the RSH, and only assuming that there is scaling for generalized structure functions of arbitrary orders. The corresponding generalized dimensions D_q^ω can be expressed through D_q, using the transformation formula (30), with $m = \frac{1}{2}$, and thus recovering Eqs. (10) and (11). Of course, this does not mean that Eq. (33) replaces the RSH. The thing here is that formula (33) applies only to longitudinal gradients, and so does the transfer formula (30), whereas the RSH incorporates full dissipation. Nevertheless, this link may explain quite a substantial correlation between the velocity increments and the pseudodissipation field, whereas this correlation is less if one considers the transverse component of the dissipation tensor [10(d)], see also Figs. 1 and 2 of Ref. [13].

In the general case, using measure (29), we write

$$\Delta_r v = \sqrt{C_2} V_n (\langle \varepsilon \rangle r)^{1/3} \langle \mu_r^{3p} \rangle^{-1/3} \mu_r^p . \tag{35}$$

In order to take into account the asymmetry, we write

$$S_q^\pm(r) \sim r^{(1-\kappa)q-(D-D_q^{\omega\pm})(q-1)}, \tag{36}$$

instead of Eq. (33) [6]. Direct measurements of $D_q^{\omega\pm}$ do confirm inequality (22), but for the dimensions $D_q^{\omega\pm}$, that is,

$$D_q^{\omega-} < D_q^{\omega+}$$

[9]. This inequality is transferred back to Eq. (22) via transformation formula (30). The formulation of the RSH is now generalized, incorporating asymmetry,

$$\Delta_r v^\pm = \sqrt{C_2} V_n (\langle \varepsilon \rangle r)^{1/3} \langle (\mu_r^-)^{3p^-} \rangle^{-1/3} (\mu_r^\pm)^{p^\pm}; \tag{37}$$

cf. Eq. (35). Note that the Kolmogorov law in this expression is satisfied only for negative distribution: the only thing that one has to make sure of is to keep the positive exponent $\zeta_3^+ \geqslant \zeta_3^- = 1$ [6]. If expression (37) is indeed adequate, then different measures for positive and negative V, as in Eqs. (34), and (32), should rectify the asymmetry for the V distribution, so that the odd moments are negative.

X. DISCUSSION AND CONCLUSION

Of course, the modification of the RSH, given by Eqs. (29) and (37), and satisfying inequalities (32), should be probed by direct measurements of the dissipation field; this is in our plans to do. The suggestions given in Sec. IX can be considered only guesses, although modifications (35) and (37) are in the spirit of the RSH, or, at least, ''not worse'' than the modification suggested in Ref. [21].

Experimental measurements of the velocity difference PDF always showed that the PDF core is almost symmetric; see, e.g., Ref. [24]. Sometimes, even an opposite symmetry is observed; for example, as noted in Ref. [24], the skewness is positive for the core. Thus the conclusion in Ref. [9], that the tails of the PDF make the main contribution to forming the Kolmogorov law, is not at all surprising. If that is the case, then, as mentioned in Secs. I and VIII, the V and y fields should correlate, as is indeed the case: this is manifested in positive $\langle V^n \rangle$, where n is an odd number.

On the other hand, however, the positive third moment $\langle V^3 \rangle > 0$ does look surprising. In most previous papers is was observed to be negative, although there were some exceptions. For example, the skewness for V does become positive in some range of distances in Fig. 8(c) of Ref. [10(e)], and some indications of positive skewness can be seen also in Fig. 3(a) of Ref. [15] (presumably, outside the inertial range, however). In our measurements, the third moment is certainly positive, at least, for some range of distances; see Fig. 7. When it is negative, it is still substantially larger than $-\frac{4}{5}$. This was confirmed by different approaches to obtaining the third moment, listed in Sec. VIII; the third moment from the pipe turbulence behaves similarly to that from run B. We cannot exclude the possibility that this difference from the other two experimental measurements, and direct numerical simulations (DNS), is an artifact of the pseudodissipation field (2); we use in our calculations. It seems more likely, though, that this difference can be explained as follows. As noted in Sec. VIII, the odd moments systematically decrease with distance, and do become negative. At least, at very large distances they should be negative. This was explained by the observation that the singularity of the dissipation field decreases with distance much faster than the singularity of the velocity increments. Therefore, we may expect that these two ranges—of positive and negative odd moments—would appear if the data provide a large enough appropriate scale range in the first place. In other words, the region where the odd moments of V are positive appears only if the Reynolds number is large enough, and that is provided by the atmospheric turbulence data. In addition to that, we note that $\langle V^3\rangle > 0$ *does not* contradict the Kolmogorov law. As mentioned, this only indicates that there is some correlation between the V field and the dissipation field, so that these two random processes are not completely statistically independent.

Substantially high correlation between the velocity increments and the dissipation field, reported in all previous papers and in the present one, implies that the RSH does work, although some deviations from it were observed. In particular, studying the asymmetry aspects of the RSH, we have to conclude that a self-consistent treatment of the asymmetry in the framework of the RSH should involve some modifications of the theory.

Further experimental studies are needed to confirm (or discard) inequality (28). This actually means that the strength of the singularity for the dissipation field corresponding to negative parts of the velocity increments is higher than that corresponding to positive parts; that is, it means that inequality (22) is satisfied, which is the very essence of the ramp model [6]. However, even independently of the ramp model, it is apparent for now that studying the asymmetry of turbulence provides some additional insight into the problem of intermittency.

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